Exercise 44

For $f(x) = \dfrac{2}{x} - \dfrac{1}{\ln x}$ find each of the following limits.

(a) $\lim\limits_{x\to\infty} f(x)$

(b) $\lim\limits_{x\to 0^+} f(x)$

(c) $\lim\limits_{x\to 1^-} f(x)$

(d) $\lim\limits_{x\to 1^+} f(x)$

(e) Use the information from parts (a)–(d) to make a rough sketch of the graph of f.

Solution

Evaluate each of the limits by plugging in the numbers.

(a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \left(\dfrac{2}{x} - \dfrac{1}{\ln x}\right) = \dfrac{2}{\infty} - \dfrac{1}{\infty} = 0 - 0 = 0$

(b) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \left(\dfrac{2}{x} - \dfrac{1}{\ln x}\right) = \infty - \dfrac{1}{-\infty} = \infty + 0 = \infty$

(c) $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \left(\dfrac{2}{x} - \dfrac{1}{\ln x}\right) = \dfrac{2}{1} - \dfrac{1}{-0} = 2 + \infty = \infty$

(d) $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \left(\dfrac{2}{x} - \dfrac{1}{\ln x}\right) = \dfrac{2}{1} - \dfrac{1}{+0} = 2 - \infty = -\infty$